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PREDICTING AND CONTROLLING RESIDUAL CHLORINE IN COOLING TOWER BLOWDOWN

G. R. Nelson

Pacific Northwest Environmental Research Laboratory
Corvallis, Oregon

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Predicting And Controlling Residual Chlorine In Cooling Tower Blowdown

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16. Abstract

A mathematical model which predicts and controls residual chlorine levels in cooling tower blowdown is developed and analyzed. The model has eight variations to allow for a) the fraction of the recirculating water chlorinated, b) the type of chlorine feed equipment used, and c) the time length of the chlorine feed period.

The variations to the model are useful not only in predicting residual chlorine levels in the blowdown, but also in making alterations in existing chlorination programs which minimize chlorine waste, provide more disinfecting efficiency, and reduce residual chlorine levels in the blowdown. (EPA)

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PREDICTING AND CONTROLLING RESIDUAL CHLORINE IN COOLING TOWER BLOWDOWN

By

Guy R. Nelson
National Thermal Pollution Research Program
Pacific Northwest Environmental Research Laboratory
National Environmental Research Center
Corvallis, Oregon

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ABSTRACT

A mathematical model which predicts residual chlorine levels in cooling tower blowdown streams at any time during the chlorination cycle is developed and analyzed in this paper. To quantify the absence or presence of residual chlorine in the blowdown, the model interprets residual chlorine as negative chlorine demand.

The general model has eight variations applying to specific chlorination program characteristics. The program characteristics affecting the general model are:

a. Split stream vs. no split stream chlorination (the fraction of the recirculating water chlorinated).

b. Residual data feedback vs. no residual data feedback (the type of chlorine feed equipment used).

c. Positive vs. negative demand at the end of the chlorine feed period. (The time length of the chlorine feed period.)

The variations to the model are useful not only in predicting residual chlorine levels in the blowdown, but also in making alterations in existing chlorination programs which minimize chlorine waste, provide more disinfecting efficiency, and reduce residual chlorine levels in the blowdown.

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SECTION I

CONCLUSIONS

The models which are developed and analyzed in this paper predict the value of RATIO during the chlorination cycle of a cooling tower system. RATIO is defined as the ratio of the blowdown chlorine demand value at any time during the chlorination cycle to its value at the start of the chlorination cycle. A positive value of RATIO indicates that there is no residual chlorine present in the blowdown. A negative value of RATIO indicates that residual chlorine is in the blowdown. RATIO's rate of decrease and increase during the chlorination cycle, and its minimum value at the end of the chlorine feed period are all a function of the cooling tower and chlorination program characteristics of specific systems.

The analysis of the models shows potential methods of reducing or eliminating residual chlorine levels in the blowdown by taking advantage of some of the optional program characteristics. These potential methods include one or more of the following alternatives:

a) Installing residual data feedback equipment into the chlorine feed system.

b) Practicing split stream chlorination.

c) Reducing the chlorine feed period, if possible.

d) Reducing the initial residual chlorine level in the condenser effluent.

e) Increasing the water volume of the cooling tower. This alternative may not apply to existing cooling towers because it involves the system design. The alternative can apply to systems

on the engineering drawing boards. This alternative may have other advantages--such as an extra supply of water for fire protection.

f) Cutting off the blowdown when residual chlorine appears in the sump. The blowdown flow can resume after the residual is dissipated by the flashing effect and the makeup water chlorine demand. The length of time during which the blowdown can be eliminated is a function of the system's upper limit on dissolved solids.

g) Mixing the blowdown with another stream which has a high chlorine demand.

To date there is no accurate field verification of the mathematical models described in this paper. For field verification it is necessary to use the amperometric procedure to measure low levels of residual chlorine (5). The amperometric procedure for determining residual chlorine in aqueous solutions is not applicable to all cooling tower waters. Some cooling tower waters contain copper, turbidity, natural buffering, and water treatment chemicals. These constituents may produce interferences in the analytical procedure which result in erroneous residual chlorine readings. In cases where limited field analyses have been performed on waters amenable to amperometric testing, trends are identified which conform to model predictions.

Even without substantial field verification, the models have utility. They can be used to modify existing and proposed chlorination programs in cooling tower systems. The modifications can provide increased chlorination efficiency and reduced residual chlorine levels in the blowdown.

SECTION II

RECOMMENDATIONS

The predictive models in this paper require substantial field verification on a broad range of cooling towers and makeup water qualities. Input from field verification can provide adjustments to the models on such factors as:

1. Incomplete mixing in the system.
2. Time dependence of chlorine demand.
3. Nonequilibrium chlorine demand.

In addition to field verification of the model, an effort is needed to improve the amperometric procedure for determining residual chlorine in problem waters. Some cooling tower waters contain varying amounts of chloramines, copper, and color which can produce interferences in the amperometric testing procedure (11). These interferences need to be averted for accurate chlorine measurement.

SECTION III

INTRODUCTION

Many cooling tower systems use chlorine or hypochlorites to control bacteria and slime growth in the condenser tubes carrying cooling water in the plant process. The biological growth, if left uncontrolled, causes excessive tube blockages, poor heat transfer, and accelerated system corrosion--all of which reduce plant efficiency. For any cooling tower system the length of time of the chlorine feed period and the number of chlorine feed periods per day, week, or month change as the biological growth problem changes. In most cooling tower systems, the chlorine is added at or near the condenser inlet in enough quantity to produce a free residual chlorine level of 0.1-0.6 mg/l in the water leaving the condenser. The amount of chlorine added to maintain the free residual chlorine depends upon the amount of chlorine demand agents and ammonia in the water.

Chlorine and ammonia react to form chloramines. These chloramines constitute the combined residual chlorine of the water. This combined residual chlorine is less efficient and slower in providing biological control than free residual chlorine (1, 2, 3, 4).

Although chlorination is effective for slime control in the condenser tubes of cooling tower systems, its application may result in residual chlorine in the blowdown discharged to the receiving water. The effects of residual chlorine on aquatic life are of great concern (1, 5, 6, 7). Data on the toxicity of residual chlorine to aquatic organisms is available. W. A. Brungs' publication, "Effects of Residual Chlorine on Aquatic Life: Literature Review",

recommends criteria for maximum residual chlorine concentrations in receiving waters (5). For the intermittent presence of residual chlorine, not to exceed two hours per day, the criteria indicate a maximum tolerable concentration of 0.2 mg/l. For the continuous presence of residual chlorine, the maximum tolerable concentration is 0.01 mg/l. These concentrations would not protect trout, salmon, and some important food organisms and are potentially lethal to sensitive life stages of sensitive fish species. Brungs recommends a lower criteria to protect trout and salmon. These lower criteria are 0.04 mg/l for the intermittent presence (2 hrs/day) of residual chlorine and 0.002 mg/l for the continuous presence of residual chlorine.

The potential effects of residual chlorine on aquatic organisms require the measurement, prediction, and control of residual chlorine in effluent discharges to the aquatic environment. The purpose of this paper is to develop and analyze a model which predicts levels of residual chlorine in the recirculation systems and blowdown streams of cooling towers. The model can be used to improve chlorination programs resulting in less chlorine waste, more disinfecting efficiency and reduced environmental impact. Although the paper is directed toward the application of chlorine in condenser cooling systems of thermal electric power plants, the model presented can be used in other industrial chlorination programs where the conditions are similar.

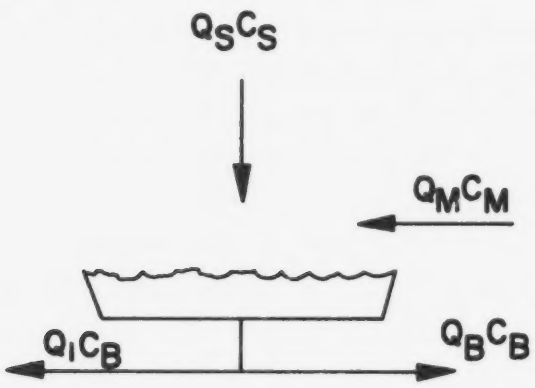
SECTION IV

MODEL DEVELOPMENT

Mass balance equations of the cooling tower system are the bases for the model development in this section. The vocabulary, concepts, and notations herein are defined in the Glossary Section. Figure 1 is a schematic of the water flows to and from the cooling tower sump.

Figure 1

Cooling Tower Sump Flows



From Figure 1, the chlorine demand mass balance around the cooling tower is,

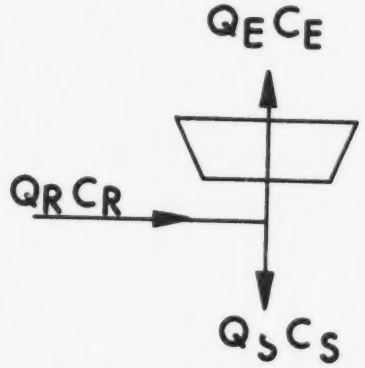
$$V\frac{dC_B}{dt} = Q_M C_M + Q_S C_S - Q_I C_B - Q_B C_B \qquad (1)$$

Equation (1) assumes that good mixing within the system causes the chlorine demand in Q_I and Q_B to be the same as the chlorine demand in the sump.

Figure 2 is a schematic of the water flows to and from the top of the cooling tower.

Figure 2

Water Flows at the Top of the Tower



From Figure 2, the chlorine demand mass balance is,

$$Q_R C_R = Q_E C_E + Q_S C_S$$

or

$$Q_R C_R - Q_E C_E = Q_S C_S \qquad (2)$$

A mass balance around the entire cooling tower system is,

$$Q_L + Q_M = Q_B + Q_E \qquad (3)$$

Just prior to the start of the chlorine feed period ($Q_L = 0$), the chlorine demand mass balance is,

$$Q_M C_M = Q_B C_{BO} + Q_E C_{EO}$$

Where C_{EO} is the chlorine demand in the evaporated water.

The model assumes that positive chlorine demand is not evaporated or flashed from the tower.

Just prior to the chlorine feed period, the chlorine demand in the system is positive and $C_{EO} = 0$; therefore,

$$Q_M C_M = Q_B C_{BO} \tag{4}$$

A mass balance around the condenser section during chlorination is,

$$Q_I + Q_L = Q_R \tag{4A}$$

The model assumes that Q_L (the flow rate of chlorine) is negligible compared with Q_I and Q_R; thus,

$$Q_I = Q_R \tag{4B}$$

By the substitution of Equations (2), (4), and (4B) back into the sump mass balance, Equation (1) becomes

$$V\frac{dC_B}{dt} = Q_B C_{BO} + Q_R C_R - Q_E C_E - Q_R C_B - Q_B C_B \tag{5}$$

Equation (5) is the general model. It is the base for determining the alternate models which apply to specific chlorination programs.

MODEL NNN

One alternate model is model NNN. This model applies to a chlorination program which: (1) does not practice split stream chlorination, (2) does not have residual feed back, and (3) has a negative demand (residual chlorine) in the blowdown at the end of the chlorine feed period.

TIME STEP A

During the chlorine feed period, residual chlorine (negative demand) is in the condenser effluent. The chlorine demand relationship at the condenser with the start of chlorination is

$$Q_R C_{BO} + Q_L C_L = Q_R C_{RO} \quad (6)$$

and at any time during the feed period it is,

$$Q_R C_B + Q_L C_L = Q_R C_R \quad (7)$$

Subtracting Equation (7) from Equation (6) results in

$$Q_R C_{BO} - Q_R C_B = + Q_R C_{RO} - Q_R C_R$$

or

$$C_R = C_{RO} - C_{BO} + C_B \quad (8)$$

The model expresses residual chlorine as negative chlorine demand. Negative demand enters the condenser section via Q_L. The term C_R thus is treated as a negative demand input in the model equation.

A portion of this negative demand (residual chlorine) flashes off or otherwise degrades in the tower.

The flashed portion is expressed in the model as $Q_E\ C_E$. Equation (9) expresses the $Q_E\ C_E$ term as a fraction of the negative demand returning to the tower.

$$F = \frac{Q_E\ C_E}{Q_R\ C_R} \quad \text{or} \quad FQ_R\ C_R = Q_E\ C_E \tag{9}$$

During Time Step A of model NNN, the general model expression is revised to,

$$V\frac{dC_B}{dt} = Q_B\ C_{BO} + (Q_R\ C_R - F\ Q_R\ C_R) - Q_R\ C_B - Q_B\ C_B$$

or

$$V\frac{dC_B}{dt} = Q_B\ C_{BO} - Q_B\ C_B - Q_R\ C_B + Q_R\ (1-F)\ (C_{RO} - C_{BO} + C_B)$$

$$= Q_B\ C_{BO} + Q_R\ (1-F)\ (C_{RO} - C_{BO}) - (Q_B + Q_R\ F)\ C_B$$

This is a first order differential equation with the solution,

$$\text{RATIO} = \frac{C_B}{C_{BO}} = X_1\ (1 - e^{-Y_1 t}) + e^{-Y_1 t} \tag{11}$$

Where

$$X_1 = \frac{Q_B/Q_R + (1-F)\left(\frac{C_{RO}}{C_{BO}} - 1\right)}{Q_B/Q_R + F}$$

and

$$Y_1 = \frac{Q_B/Q_R + F}{V/Q_R}$$

Equation (11) expresses the value of RATIO during the chlorine feed period of model NNN. In the model, the ratio starts at unity and decreases as TSA continues. At the end of TSA, RATIO is negative. This indicates the presence of residual chlorine in the blowdown. The concentration of residual chlorine (mg/l) in the blowdown is found by multiplying RATIO by C_{BO}.

RATIO $\cdot$ C_{BO} = Residual Chlorine.

The time symbol, (t), in Equation (11) represents the time during TSA (i.e. at the start of TSA, t=0).

TIME STEP B

Time step B (TSB) defines the length of time which RATIO remains negative after the end of TSA. The model expression for TSB, therefore, requires a negative value of RATIO at the start of TSB. The symbol, $(C_B/C_{BO})_B$, represents the value. During TSB, no chlorine is added, thus, $C_R = C_B$.

Since there is still residual chlorine in the water returning to the tower, the flashing term still applies in the mass balance.

The general model expression during TSB is

$$V\frac{dC_B}{dt} = Q_B\, C_{BO} - Q_B\, C_B - Q_R\, C_R + Q_R\,(1-F)\, C_B$$

or

$$V\frac{dC_B}{dt} = Q_B\, C_{BO} - Q_B\, C_B - Q_R\, F\, C_B$$

This is a first order differential equation with the solution,

$$\text{RATIO} = \frac{C_B}{C_{BO}} = X_2\,(1-e^{-Y_2 t}) + (C_B/C_{BO})_B\, e^{-Y_2 t} \qquad (12)$$

t = 0 at the start of TSB

$$\text{Where } X_2 = \frac{Q_B/Q_R + F}{Q_B/Q_R}$$

$$Y_2 = \frac{Q_B/Q_R + F}{V/Q_R}$$

and $(C_B/C_{BO})_B$ = The value of RATIO at the start of TSB.

Equation (12) is the mathematical expression for TSB of models NNN, NRN, SNN, and SRN. By definition models NNP, NRP, SNP, and SRP do not have TSB in the chlorination cycle.

TIME STEP C

During TSC there is no negative chlorine demand (residual chlorine) entering the cooling tower sump. The chlorine demand mass balance around the sump is therefore,

$$V\frac{dC_B}{dt} = Q_B\, C_{BO} - Q_B\, C_B$$

This is a first order differential equation with the solution:

$$\text{RATIO} = 1 - [1 - (C_B/C_{BO})]\; e^{-\frac{Q_B t}{V}} \qquad (13)$$

Where t = 0 at the start of TSC and $(C_B/C_{BO})_C$ is the value of ratio at the start of TSC. Equation (13) is the mathematical expression for TSC of all the models.

For models NNN, NRN, SNN, and SRN, $(C_B/C_{BO})_C$ is zero so Equation (13) is reduced to,

$$\text{RATIO} = 1 - e^{-\frac{Q_B t}{V}} \qquad (14)$$

Equation (14) is the mathematical expression covering TSC of models NNN, NRN, SNN, and SRN.

Appendix A contains the mathematical expressions for all of the time steps of the eight models.

Appendix B contains the derivations of the expressions for TSA of models NRN, NRP, SRN, SRP, SNN, and SNP.

SECTION V

MODEL ANALYSIS

The purpose of this section is to analyze and discuss the models developed in the previous section. The purpose is accomplished by detailing three chlorination program studies on two cooling tower systems. One program is for a mechanical draft cooling tower system; the other is for a natural draft cooling tower system.

Six cooling tower characteristics affect the value of RATIO during the chlorination cycle in both cooling tower systems.

a) The cooling system volume (V)
b) The recirculation rate (Q_R)
d) The initial chlorine demand in the blowdown (C_{BO})
e) The flashing rate of residual chlorine to the atmosphere (F)
f) The initial residual chlorine level in the condenser effluent
g) The blowdown flow rate (Q_B)

These characteristics manifest themselves in the predictive models through the terms discussed below.

The V/Q_R term (expressed in minutes) is the ratio of the water volume of the entire cooling system to the recirculating water flow rate. This term, indicates the length of time for a given water parcel to make one pass through the system. With conventional system design, this value is 10-15 minutes for mechanical draft towers and 20-22 minutes for natural draft towers.

The dimensionless term, Q_B/Q_R is the ratio of the blowdown water flow rate to the recirculating water flow rate. Its value, along with the V/Q_R term, expresses the length of time that a conservative chemical remains in the system.

The dimensionless term, C_{RO}/C_{BO}, is the ratio of the initial residual chlorine level in the recirculating water (C_{RO}) to the initial chlorine demand in the blowdown (C_{BO}).

The dimensionless term, F, is the fraction of the residual chlorine which flashes or decomposes as it passes through the cooling tower fill. The theoretical range of the value of F can be 0.0-1.0. Probably F is related in some yet unquantified way to tower inlet temperature and to cooling range and/or the water to air ratio; the chemical form of the chlorine residual may also be a factor. No field verified value of F is available; however, its existence is documented (1,3,9). In the draft Environmental Impact Statement for the Davis Besse steam electric generating plant, Draley suggests an F value of 0.5 for combined residual chlorine in the natural draft tower system (9).

Table 1 lists the values of cooling tower and chlorination program characteristics found in typical cooling tower systems. The table also lists the values which will be applied to the two hypothetical cooling tower programs.

TABLE 1

Cooling Tower and Chlorination Program Characteristics
(Ref. 1, 2, 3, 4, 8, 9, 10)

Characteristic	Typical Values		Values Applied To Program Studies
	Min	Max	
A. Chlorination Program:			
Chlorination cycle	8 hrs	7 days	24 hr
Chlorine feed period	10 min	30 min	15 min
Split stream Cl_2	Optional		Program Study 3
Residual Feedback	Optional		Program Study 2
B. Cooling Tower:			
Q_B/Q_R	0.008	0.015	0.01
F	0.3	0.6	0.4
C_{RO}	-0.1	-1.0	-0.4
C_{BO}	0.5	3.0	0.667
C_{RO}/C_{BO}	-0.1	-1.2	-0.6
V/Q_R*	10	15	10
V/Q_R**	20	22	20
C. Model Constant (calculated from B. above):			
X_1			-2.32
Y_1*			0.041 min^{-1}
Y_1**			0.0205 min^{-1}
X_2			0.0244

* Mechanical draft
** Natural draft

PROGRAM STUDY ONE

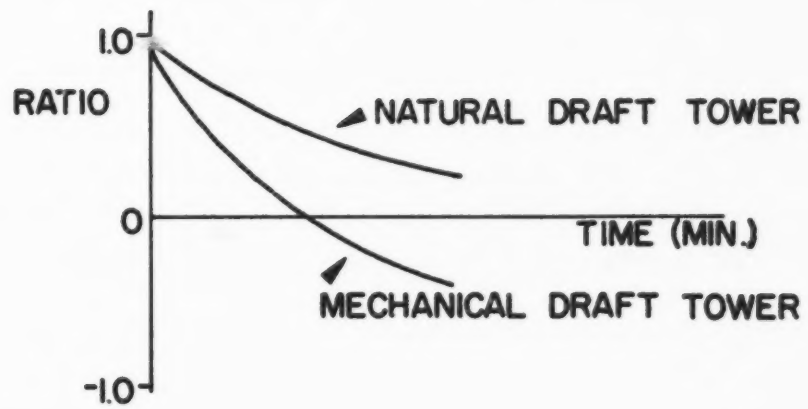
In this first study, the chlorination program characteristics do not contain residual data feedback or split stream chlorination. From Appendix A, the expression for TSA of models NNN and NNP is

$$RATIO = X_1 (1-e^{-Y_1 t}) + e^{-Y_1 t}$$

Figure 3 is a graphical representation of the model during TSA for both programs. In the natural draft program, RATIO remains positive throughout the chlorine feed period. This indicates that there is no residual chlorine in the blowdown.

Figure 3

RATIO During TSA



In the mechanical draft program, RATIO becomes negative during TSA. The boundary condition, t = 15 min., determines its value at the end of TSA.

$$RATIO = -2.32 (1-e^{-0.041 (15)}) + e^{-0.041 (15)}$$

$$= -0.523$$

The chlorine residual in the blowdown at the end of TSA is,

$$Cl_2 \text{ residual} = 0.523\ (0.667 \text{ mg/l})$$

$$= 0.349 \text{ mg/l}$$

The first appearance of residual chlorine in the blowdown occurs when there is zero chlorine demand in the sump, or RATIO = 0. The time during the chlorine feed period when this occurs is,

$$\text{RATIO} = 0 = e^{-0.041t} - 2.32\ (1 - e^{-0.041t})$$

$$t = 24.4\ [\ln\ (1.432)] \text{ minutes}$$

$$= 8.75 \text{ min}$$

From the above calculations, residual chlorine is present in the blowdown for 6.25 minutes during TSA.

A residual first appears at 8.75 minutes after the start of the chlorine feed period. This residual increases in value until the end of the feed period, at which time it is at its maximum value. Since the model defines residual chlorine as negative chlorine demand, RATIO is at its minimum value at this time.

TIME STEP B

The TSB expression does not apply to the natural draft program, because its terminal demand value is not negative. The TSB expression for the mechanical draft program is,

$$RATIO = X_2 (1-e^{-Y_2 t}) + (C_B/C_{BO})_B \, e^{-Y_2 t}$$

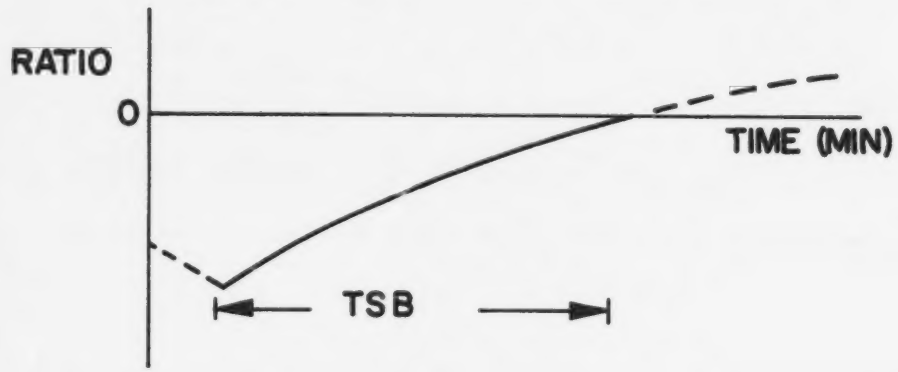
Figure 4 is a graphical representation of TSB. The value of RATIO at the start of TSB is,

$$RATIO = (C_B/C_{BO})_B = -0.523$$

RATIO increases through TSB until it reaches the value of zero.

Figure 4

Time Step B for the Mechanical Draft Program



The boundary condition, RATIO = 0, at the end of TSB quantifies the time of TSB:

$$0 = 0.0244 (1-e^{-0.041t}) - (0.523) e^{-0.041t}$$

$$t = 24.4 \text{ in } (22.4) \text{ min}$$

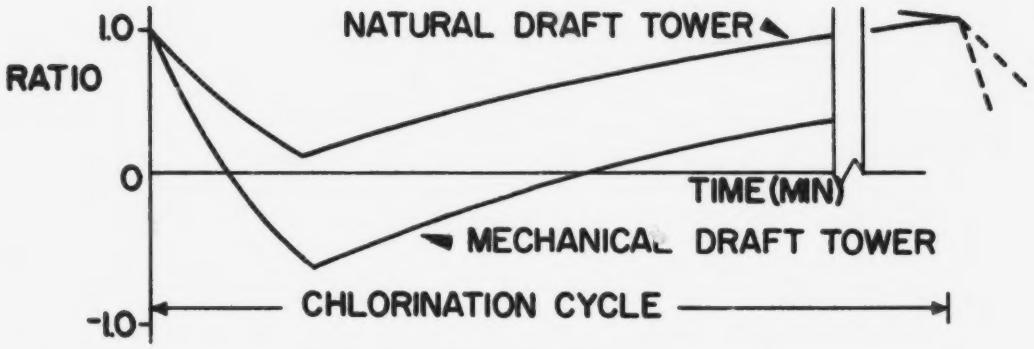
$$= 76 \text{ min}$$

The mechanical draft program in this study causes residual chlorine in the blowdown to occur for nearly 83 minutes (76 + 6.25) during the chlorination cycle.

Figure 5 illustrates the value of RATIO through the complete chlorination cycle for both programs under program study one.

Figure 5

Study One RATIO Values



In order to minimize repetition, the following two program studies will concentrate on alterations to the mechanical draft program. The concepts in program studies two and three are applicable to natural draft programs.

PROGRAM STUDY TWO

This program study analyzes the effect of residual feedback on the value of RATIO for the mechanical cooling tower draft. Either model NRN or NRP applies in this case--depending on the value of RATIO at the end of TSA.

From Appendix A, the expression for models NRN and NRP during TSA is,

$$RATIO = \frac{Q_B/Q_R + (1-F)\, C_{RO}/C_{BO}}{Q_B/Q_R + 1} \left(1-e^{-Y_5 t}\right) + e^{-Y_5 t}$$

$$\text{Where } Y_5 = \frac{Q_B/Q_R + 1}{V/Q_R}$$

From the values of Table 1, this reduces to,

$$RATIO = -0.346 \left(1-e^{-0.101t}\right) + e^{-0.101t}$$

The value of RATIO at the end of the chlorine feed period is

$$RATIO = -0.346 \left(1-e^{-1.515}\right) + e^{-1.515}$$

$$= -0.27 + 0.22$$

$$= -0.05$$

The negative value of RATIO indicates the presence of residual chlorine. Its value is

$$\text{Residual chlorine} = 0.05\ (0.667)\ \text{mg/l}$$

$$= 0.033\ \text{mg/l}$$

The time during TSA in which a residual appeared is

$$RATIO = 0 = -0.346 \left(1-e^{-0.101t}\right) + e^{-0.101t}$$

$$t = 9.9 \ln 3.89 \text{ min}$$

$$= 13.45 \text{ min}$$

Since the value of RATIO is negative, TSB of model NRN for this case is

$$RATIO = 0.0244\ (1-e^{-0.101t}) - 0.05\ e^{-0.101t}$$

The time required for RATIO to recover to zero is,

$$t = 9.9 \ln (3.05) \text{ min}$$

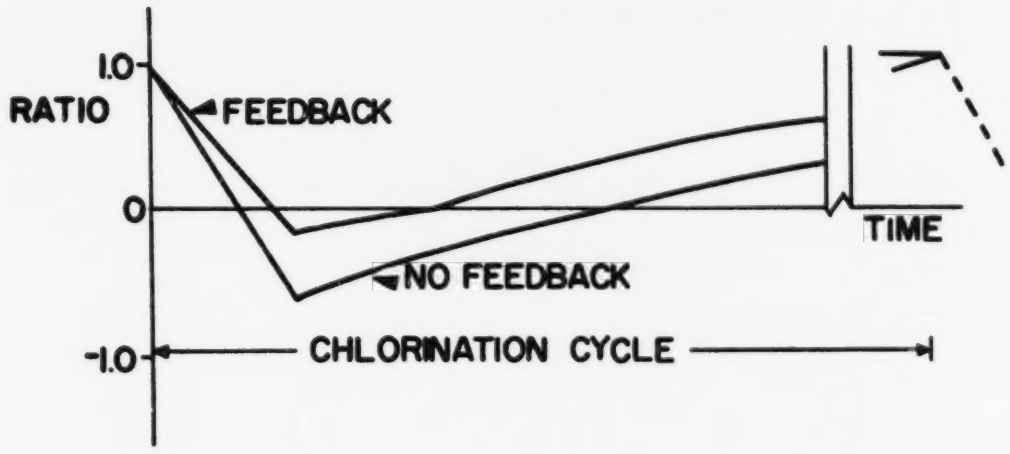
$$= 13.4 \text{ min}$$

In this case residual chlorine is in the blowdown for about 15 min. Its highest value during this time is 0.033 mg/l.

Figure 6 compares the chlorination cycle in this study to the cycle for the mechanical draft program which did not have residual data feedback.

Figure 6

Residual Data Feedback vs. No Residual Data Feedback



The comparison of the two studies shows a marked reduction in the concentration of residual chlorine in the blowdown of the cooling tower in study two. The use of residual data feedback in the mechanical draft program not only reduces the concentration of residual chlorine in the blowdown, but also reduces the length of time during the chlorination cycle in which the blowdown contains residual chlorine.

PROGRAM STUDY THREE

In this program study, the chlorination program for the mechanical draft tower includes split stream chlorination and excludes residual data feedback. In this case, either SNN or SNP applies--depending on the value of RATIO at the end of TSA.

The TSA expression in both models is (from Appendix A),

Substep 1

$$\text{RATIO} = \left[\frac{Q_B/Q_R + S\left(\frac{C_{TO}}{C_{BO}} - 1\right)}{Q_B/Q_R}\right]\left(1-e^{-Y_6 t}\right) + e^{-Y_6 t}$$

Substep 2

$$\text{RATIO} = \left[\frac{Q_B/Q_R + (1-F)\,S\left(\frac{C_{TO}}{C_{BO}} - 1\right)}{Q_B/Q_R + F}\right]\left(1-e^{-Y_7 t}\right) + S\left(1 - \frac{C_{TO}}{C_{BO}}\right) e^{-Y_7 t}$$

Where S is the fraction of the recirculating water chlorinated and C_{TO} is the initial residual of the split stream.

There are two substeps to TSA in models SNN, SNP, SRN, and SRP. The first substep applies until there is residual chlorine in the recirculating water after the split stream is remixed with the remaining streams. The flashing term (F) does not appear in the substep 1 expression because the chlorine demand is positive (no residual chlorine). Negative demand appears in the recirculating water when RATIO = S $(1 - C_{TO}/C_{BO})$. Once the condition is reached, then the substep 2 expression applies for the rest of TSA.

If the chlorination program calls for the chlorination of 50 percent of the recirculating water at an initial residual chlorine level of 0.4 mg/l, the substeps for this study are:

Substep 1

$$RATIO = (-79)\ (1-e^{-0.001t}) + e^{-0.0.001t}$$

Substep 2

$$RATIO = -1.15\ \ (1-e^{-0.041t}) + 0.8\ e^{-0.041t}$$

To determine the time length of substep 1, RATIO is set equal to the boundary condition, $-S\ (C_{TO}/C_{BO} -1)$, = 0.8 and the expression is solved for t.

$$0.8 = -79\ (1-e^{-0.001t}) + e^{-0.001t}$$

$$t = 1000\ \ln\ (1.0025)\ \text{min}$$

$$= 2.5\ \text{min}$$

The length of substep 2 is

$$15 - 2.5 = 12.5 \text{ min}$$

The value of RATIO at the end of TSA is

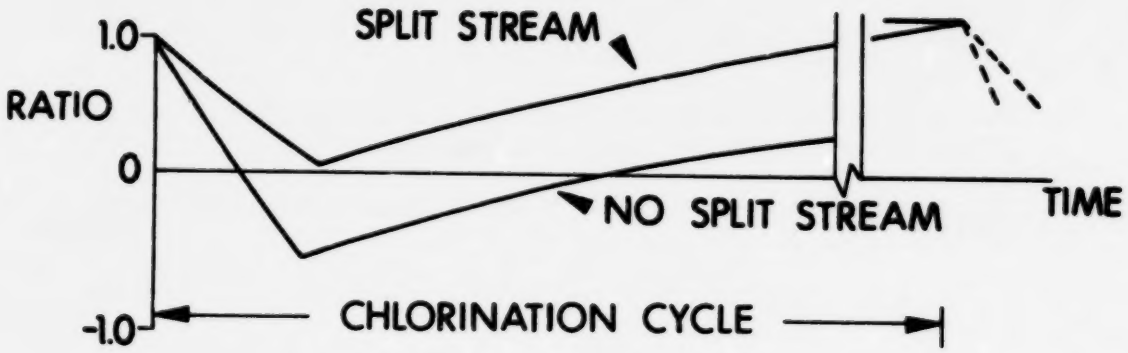
$$\text{RATIO} = -1.15\,(1-e^{-0.041\,(12.5)}) + 0.8\,e^{-0.041\,(12.5)}$$

$$= +0.02$$

The positive demand value at the end of TSA indicates that there is no residual chlorine in the blowdown. TSB does not apply; therefore, RATIO regenerates back to the value of positive unity before the start of the next chlorine feed period. Figure 7 compares the chlorination cycle of this study to the cycle of the mechanical draft program of study one which does not contain split stream chlorination.

Figure 7

Split Stream vs. No Split Stream Chlorination



Study three shows the value of split stream chlorination in minimizing or eliminating residual chlorine in the blowdown.

SECTION VI

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SECTION VII

GLOSSARY

VOCABULARY

Free residual chlorine is that portion of the total residual chlorine which will react chemically and biologically as hypochlorus acid or hypochlorite ion.

Combined residual chlorine is that portion of the total residual chlorine which will react chemically and biologically as chloramines.

Total residual chlorine is the sum of the free and combined residuals. Unless otherwise specified, throughout this paper residual chlorine refers to the total residual chlorine.

Chlorine demand is the amount of chlorine (mg/l) required to be added to a water (sample) before any stable residual chlorine is formed. Organics and reducing agents in the water cause this demand. These materials have varying reaction rates with chlorine. The reaction rates cause the chlorine demand value to be time dependent. For the purpose of this paper, the chlorine demand is that demand which reacts with chlorine within five minutes of exposure.

The chlorination program describes the manner in which chlorine is fed and controlled in the cooling tower system.

The chlorination cycle is the length of time between the start of two sequential chlorine feed periods.

Split Stream chlorination is an alternate method of chlorine addition. It is the practice of splitting the total recirculation flow through the condenser into a number of separate streams. One of these streams is chlorinated at a time. The chlorinated stream is then mixed with the remaining streams. The presence or absence of split stream chlorination is a chlorination program characteristic.

Residual feedback describes a function performed by chlorine feed equipment. If the control system in the equipment is capable of adjusting the flow of chlorine to produce a constant residual in the recirculation water out of the condenser, the system has residual feedback. The presence or absence of residual feedback is a chlorination program characteristic.

CONCEPTS

Residual chlorine vs. chlorine demand - The model expresses the change in the chlorine demand of the blowdown during the chlorination cycle. To quantify the absence or presence of residual chlorine in the blowdown, the model interprets residual chlorine as negative chlorine demand. By conceptual definition, residual chlorine is not present in the blowdown unless the chlorine demand is satisfied.

C_B - In the model development, the term C_B represents the chlorine demand in the blowdown at any time during the chlorination cycle.

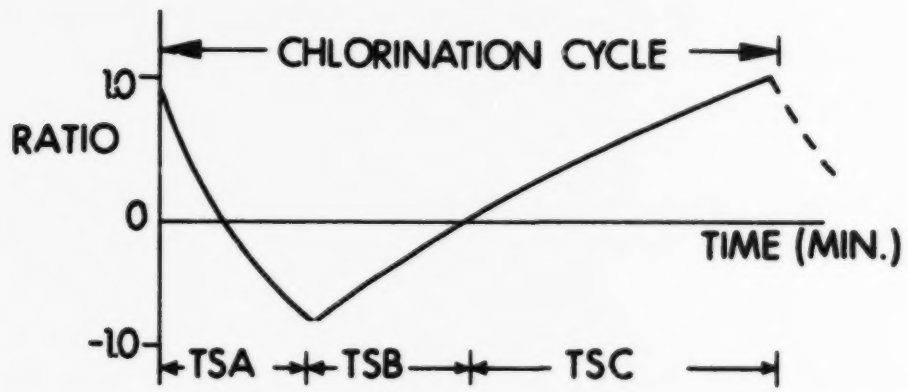
C_{BO} - The term C_{BO} represents the chlorine demand in the blowdown at the beginning of the chlorination cycle.

RATIO - The term RATIO represents the ratio of the chlorine demand in the blowdown at any time during the chlorination cycle to its initial value at the beginning of the cycle (i.e., RATIO = C_B/C_{BO}).

Time Steps - In order to predict the chlorination cycle, the model breaks down the cycle into time steps. Figure 8 illustrates the time step concept.

Figure 8

Time Steps of the Chlorination Cycle



Time Step A (TSA) - TSA is the length of time during which chlorine is added to the system (the chlorine feed period).

Time Step B (TSB) - TSB is the length of time after TSA in which the value of RATIO is negative.

Time Step C (TSC) - TSC is the length of time after TSA of a chlorination cycle in which the value of RATIO is zero or positive. TSC ends at the start of the next chlorination cycle.

Although equations can be (and have been) written for Time Step C the numerical output is rather spurious and of no practical value in either environmental protection or chlorination program design. This is true primarly because the length of Time Step C is dictated by the biocidal requirements of the cooling system rather than any level or function of chlorine demand or residual obtainable from the equations. Also, extraneous factors such as dust washout or changes in makeup water characteristics are not accounted for. Finally, optimal use of the models for environmental protection and chlorine conservation require chlorine demand data at start of the cycle (C_{BO}).

Terminal Demand Value (TDV) - TDV is the value of RATIO at the end of TSA. It is the lowest value of RATIO during the chlorination cycle.

Positive vs. Negative TDV - TDV can be either positive or negative at the end of TSA. If TDV is negative as shown in Figure 1, then the cycle contains three time steps (TSA, TSB, and TSC). If TDV is positive or zero, then the cycle contains only two time steps (TSA and TSC). The positive vs. negative TDV is a chlorination program characteristic.

MODEL NOTATION

There are eight specific models based upon the general model discussed in this paper. Each specific model applies to a set of chlorination program characteristics. Table 2 is a matrix which defines the

shorthand notation used to describe each model. For easy reference, the first letter in the shorthand notation refers to sidestream filtration or no sidestream filtration (S or N). The second letter refers to residual feedback (R or N). The third letter refers to positive TDV or negative TDV (P or N).

TABLE 2

Model Notation

	Residual Feedback		No Residual Feedback	
	(-) TDV	(+) TDV	(-) TDV	(+) TDV
Split Stream Chlorination	SRN	SRP	SNN	SNP
No Split Stream Chlorination	NRN	NRP	NNN	NNP

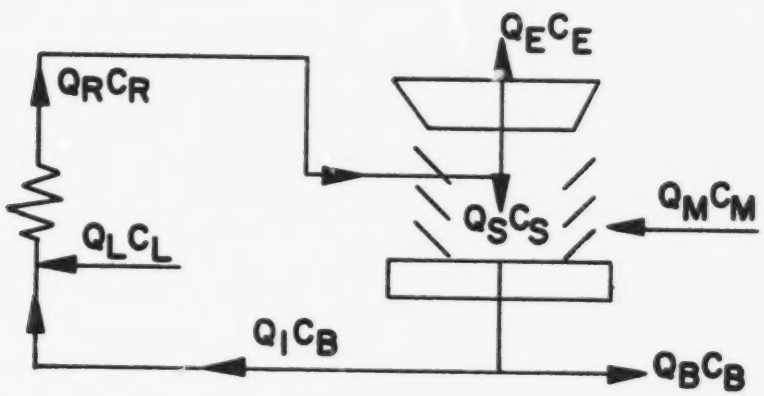
For example, model NNN applies to a chlorination program which a) does not have split stream chlorination, b) does not have residual feedback, and c) has a negative chlorine demand at the end of the chlorine feed period. Model SRP applies to a program which a) has split stream chlorination, b) has residual feedback, and c) has a positive chlorine demand at the end of the chlorine feed period.

Cooling System Notation

Figure 9 is a schematic flow diagram of a typical cooling tower system;

Figure 9

Cooling Tower System



Where

- Q_B = Blowdown flow rate in m^3/min (gpm)
- Q_I = Water flow rate into the condenser in m^3/min (gpm)
- Q_L = Chlorine flow rate into the condenser in m^3/min (gpm)
- Q_R = Water flow rate returning to tower from the condenser in m^3/min (gpm)
- Q_E = Water evaporation rate leaving tower stack in m^3/min (gpm)
- Q_S = Water flow rate to the sump in m^3/min (gpm)
- Q_M = Makeup water flow rate into the tower sump in m^3/min (gpm)
- V = Cooling tower system volume in m^3 (gallons)
- C_B = Chlorine demand in the blowdown, (mg/l)
- C_{BO} = Chlorine demand in the blowdown at the start of the chlorine feed period (mg/l)
- C_L = Chlorine demand in the chlorine feed stream (mg/l)

C_R = Chlorine demand in water returning to the tower from the condenser (mg/l)

C_{RO} = Chlorine demand in the water returning to the tower from the condenser at the start of the chlorine feed period (mg/l)

C_E = Chlorine demand in the evaporated water leaving the tower (mg/l)

C_S = Chlorine demand in the recirculating water entering the sump (mg/l)

C_M = Chlorine demand in the makeup water entering the sump

SECTION VIII

APPENDICES

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APPENDIX A

Alternate Model Expressions For Specific Chlorination Programs

TABLE 2

Models

Programs without Split Stream Chlorination				Programs with Split Stream Chlorination				
Model	Equations			Model	Equations			
	TSA	TSB	TSC		TSA		TSB	TSC
					Sub 1	Sub 2		
NNN	1A	2A	4A	SNN	6A	7A	2A	4A
NNP	1A		3A	SNP	6A	7A		3A
NRN	5A	2A	4A	SRN	8A	9A	2A	4A
NRP	5A		3A	SRP	8A	9A		3A

Equations referred to in Table A-1

(1A) $$\frac{C_B}{C_{BO}} = X_1 (1-e^{-Y_1 t}) + e^{-Y_1 t}$$

where $$X_1 = \frac{\frac{Q_B}{Q_R} + (1-F)\left(\frac{C_{RO}}{C_{BO}} - 1\right)}{\frac{Q_B}{Q_R} + F}$$

$$Y_1 = \frac{\frac{Q_B}{Q_R} + F}{\frac{V}{Q_R}}$$

(2A) $$\frac{C_B}{C_{BO}} = X_2(1-e^{-Y_2 t}) + \left(\frac{C_B}{C_{BO}}\right)_B e^{-Y_2 t}$$

where $$X_2 = \frac{\frac{Q_B}{Q_R}}{\frac{Q_B}{Q_R} + F}$$

$$Y_2 = \frac{\frac{Q_B}{Q_R} + F}{\frac{V}{Q_R}}$$

$\left(\frac{C_B}{C_{BO}}\right)_B$ = RATIO's value at the start of TSB

****

(3A) $$\frac{C_B}{C_{BO}} = 1-(1-\left(\frac{C_B}{C_{BO}}\right)_C) e^{-Y_3 t}$$

where $Y_3 = \frac{Q_B}{V}$

$\left(\frac{C_B}{C_{BO}}\right)_C$ = RATIO's value at the start of TSC

****

(4A) $$\frac{C_B}{C_{BO}} = 1-e^{-Y_4 t}$$

where $Y_4 = \frac{Q_B}{V}$

(5A) $$\frac{C_B}{C_{BO}} = X_5(1-e^{-Y_5 t}) + e^{-Y_5 t}$$

where $$X_5 = \frac{\frac{Q_B}{Q_R} + (1-F)\frac{C_{RO}}{C_{BO}}}{\frac{Q_B}{Q_R} + 1}$$

$$Y_5 = \frac{\frac{Q_B}{Q_R} + 1}{\frac{V}{Q_R}}$$

****

(6A) $$\frac{C_B}{C_{BO}} = X_6(1-e^{-Y_6 t}) + e^{-Y_6 t}$$

where $$X_6 = \frac{\frac{Q_B}{Q_R} + S(\frac{C_{TO}}{C_{BO}} - 1)}{\frac{Q_B}{Q_R}}$$

$$Y_6 = \frac{Q_B}{V}$$

C_{TO} = Chlorine demand in chlorinated condenser effluent at the start of TSA

S = Chlorinated fraction of total recirculating water flow

(7A) $$\frac{C_B}{C_{BO}} = X_7(1-e^{-Y_7 t}) + S\left(1-\frac{C_{TO}}{C_{BO}}\right) e^{-Y_7 t}$$

where $$X_7 = \frac{\frac{Q_B}{Q_R} + (1-F)S\left(\frac{C_{TO}}{C_{BO}} - 1\right)}{\frac{Q_B}{Q_R} + F}$$

$$Y_7 = \frac{\frac{Q_B}{Q_R} + F}{\frac{V}{Q_R}}$$

C_{TO} = chlorine demand in chlorinated condenser effluent at the start of TSA

S = Chlorinated fraction of total recirculating water flow

****

(8A) $$\frac{C_B}{C_{BO}} = X_8(1-e^{-Y_8 t}) + e^{-Y_8 t}$$

where $$X_8 = \frac{\frac{Q_B}{Q_R} + S\frac{C_{TO}}{C_{BO}}}{\frac{Q_B}{Q_R} + S}$$

$$Y_8 = \frac{\frac{Q_B}{Q_R} + S}{\frac{V}{Q_R}}$$

C_{TO} = chlorine demand in chlorinated condenser effluent at the start of TSA

S = Chlorinated fraction of total recirculating water flow

$$(9A)\quad \frac{C_B}{C_{BO}} = X_9(1-e^{-Y_9 t}) + \left(\frac{S}{S-1}\right)\left(\frac{C_{TO}}{C_{BO}}\right) e^{-Y_9 t}$$

where $$X_9 = \frac{\frac{Q_B}{Q_R} + (1-F)S\frac{C_{TO}}{C_{BO}}}{\frac{Q_B}{Q_R} + S+F-FS}$$

$$Y_9 = \frac{\frac{Q_B}{Q_R} + S + F - FS}{\frac{V}{Q_R}}$$

C_{TO} = chlorine demand in chlorinated condenser effluent at the start of TSA

S = Chlorinated fraction of total recirculating water flow

APPENDIX B

Derivations

MODELS SNN AND SNP

Time Step A

TSA in these models has two substeps. Substep 1 covers the time during TSA when the Q_R stream has a positive demand. This condition occurs during the time when the residual chlorine in the chlorinated split stream does not satisfy the chlorine demand of the remaining portion of the recirculating water. The boundary conditions are:

at $t = 0$, RATIO = 1

at $t = t_1$, $\text{RATIO} = -S \left(\frac{C_{TO}}{C_{BO}} - 1\right)$

S = fraction of the total recirculating water flow which is chlorinated,

and

C_{TO} = initial residual chlorine level in the split stream out of the condenser.

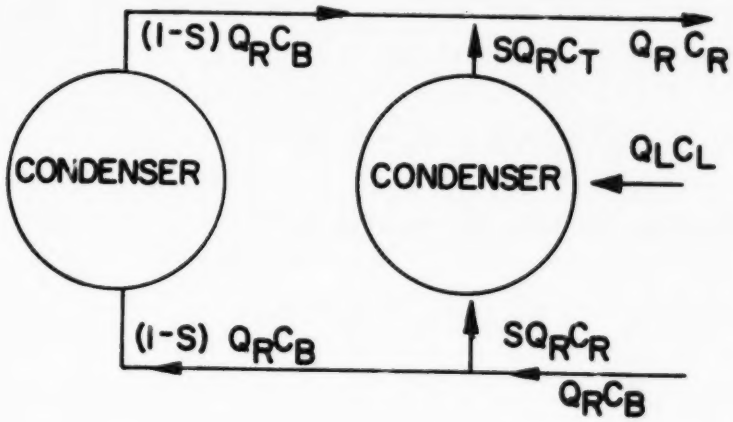
Substep 2 covers the time during TSA when the chlorine demand in the Q_R stream is negative (residual chlorine present).

Substep 1

Figure 10 is a schematic of the condenser system with split stream chlorination.

Figure 10

Split Stream Chlorination



The chlorine demand relationship at the chlorinated condenser is:

$$S\ Q_R\ C_B + Q_L\ C_L = S\ Q_R\ C_T \qquad (1)$$

At the start of the feed period it is:

$$S\ Q_R\ C_{BO} + Q_L\ C_L = S\ Q_R\ C_{TO} \qquad (2)$$

Subtracting (1) from (2) results in:

$$S\,Q_R\,C_B - S\,Q_R\,C_{BO} = S\,Q_R\,C_T - S\,Q_R\,C_{TO}$$

and

$$C_T = C_{TO} - C_{BO} + C_B \qquad (3)$$

At the point where the split stream recombines with the remainder of the flow, the relationship is:

$$(1-S)\,(Q_R)\,C_B + S\,Q_R\,C_T = Q_R\,C_R \qquad (4)$$

Substituting (3) into (4) results in:

$$(1-S)\,C_B + S\,(C_{TO} - C_{BO} + C_B) = C_R$$

or

$$C_R = C_B - S\,C_B + S\,(C_{TO} - C_{BO}) + SC_B$$

or

$$C_R = C_B + S\,(C_{TO} - C_{BO}) \qquad (5)$$

Substep 1 holds for all positive values of C_R. It terminates when:

$$t = t_1\ C_R = 0 = C_B + S\,(C_{TO} - C_{BO})$$

or

$$C_B = - S (C_{TO} - C_{BO})$$

and

$$RATIO = - S \left(\frac{C_{TO}}{C_{BO}} - 1\right)$$

During substep 1 the mass balance around the sump is:

$$V\frac{dC_B}{dt} = Q_B C_{BO} - Q_B C_B - Q_R C_B + Q_R [C_B + S (C_{TO} - C_{BO})] \qquad (6)$$

The solution of (6) is:

$$\frac{C_B}{C_{BO}} = RATIO = \frac{Q_B/Q_R + S \left(\frac{C_{TO}}{C_{BO}} - 1\right)}{Q_B/Q_R} (1-e^{-Y_6 t}) + e^{-Y_6 t} \qquad (7)$$

Equation (7) is the expression of RATIO during substep 1.

Substep 2

During substep 2 the value of C_R is negative, thus, the mass balance is:

$$V\frac{dC_B}{dt} = Q_B C_{BO} - Q_B C_B - Q_R C_B + Q_R (1-F) [S (C_{TO} - C_{BO}) + C_B]$$

With the solution:

$$RATIO = \frac{C_B}{C_{BO}} = \left[\frac{Q_B/Q_R + (1-F)\ S\ (\frac{C_{TO}}{C_{BO}} - 1)}{Q_B/Q_R + F}\right] [1-e^{-Y_7 t}]$$

$$- S(\frac{C_{TO}}{C_{BO}} - 1)\ e^{-Y_7 t} \qquad (8)$$

Equation (8) is the expression of RATIO during substep 2.

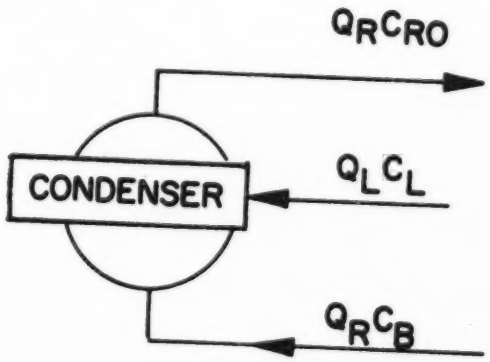
TSB and TSC for the models are derived in the main body of the paper.

MODELS NRN AND NRP

Figure 11 is a schematic of the condenser system with residual data feedback. Residual data feedback holds a constant residual chlorine level in the recirculating water out of the condenser.

Figure 11

Residual Data Feedback Chlorination



The chlorine demand relationship at the condenser during TSA is:

$$Q_R C_B + Q_L C_L = Q_R C_{RO}$$

Since the residual chlorine level out of the condenser remains constant during TSA; the expression for C_R is:

$$C_R = C_{RO}$$

and the mass balance around the sump is:

$$V\frac{dC_B}{dt} = Q_B C_{BO} - Q_B C_B - Q_R C_B + Q_R (1-F) C_{RO} \qquad (9)$$

The solution for Equation (9) is:

$$RATIO = \frac{Q_B/Q_R + (1-F)\frac{C_{RO}}{C_{BO}}}{1 + Q_B/Q_R}(1-e^{-Y_5 t}) + e^{-Y_5 t} \qquad (10)$$

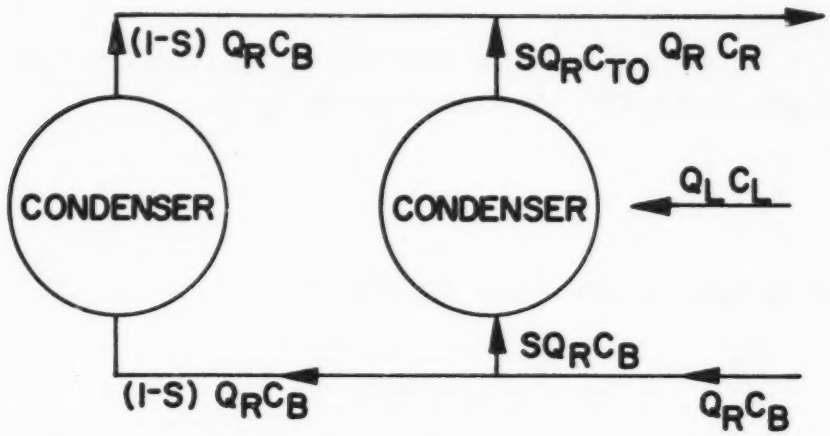
Equation (10) is the expression for models NRN and NRP during TSA.

MODELS SRN AND SRP

These two models require two substeps during TSA. Figure 12 is a schematic of the condenser flows during TSA.

Figure 12

Combination Split Stream/Residual Data Feedback Chlorination



The residual feedback mechanism holds the residual chlorine level constant in the chlorinated split stream.

Substep 1

During substep 1, the chlorine demand in the recirculating water after the split streams are rejoined is positive. The relationship at the point of juncture is:

$$(1-S)\ Q_R\ C_B + S\ Q_R\ C_{TO} = Q_R\ C_R$$

or

$$C_B - SC_B + SC_{TO} = C_R$$

$$C_R = (1-S)\ C_B + SC_{TO}$$

and the mass balance around the sump is:

$$V\frac{dC_B}{dt} = Q_B C_{BO} - Q_B C_B - Q_R C_B + Q_R ((1-S) C_B + S C_B)$$

$$= Q_B C_{BO} - Q_B C_B - Q_R S C_B + Q_R S C_{TO}$$

$$= Q_B C_{BO} + Q_R S C_{TO} - (Q_B + Q_R S) C_B$$

The solution to this equation is:

$$RATIO = \frac{Q_B/Q_R + S\frac{C_{TO}}{C_{BO}}}{Q_B/Q_R + S}(1-e^{-Y_B t}) + e^{-Y_B t} \qquad (11)$$

Equation (11) is the expression for RATIO during substep 1 of TSA. The boundary conditions for substep 1 are:

at $t = 0$, RATIO = 1

at $t = t_1$, $RATIO = \frac{-SC_{TO}}{(1-S) C_{BO}}$

Substep 2

During substep 2, the mass balance around the sump is:

$$V\frac{dC_B}{dt} = Q_B C_{BO} - Q_B C_B - Q_R C_B + Q_R (1-F) [SC_{TO} + (1-S) C_B]$$

or

$$= Q_B C_{BO} + Q_R (1-F) SC_{TO} - Q_B C_B - Q_R C_B + Q_R (1-S-F+FS) C_B$$

$$= Q_B C_{BO} + Q_R (1-F) SC_{TO} - Q_B C_B - Q_R (S+F-FS) C_B$$

The solution to this equation is:

$$RATIO = \frac{Q_B/Q_R + (1-F) S \frac{C_{TO}}{C_{BO}}}{Q_B/Q_R + S + F - FS} [1-e^{-Y_g t}] - \frac{SC_{TO}}{(1-S)C_{BO}} e^{-Y_g t} \qquad (12)$$

Equation 12 is the expression for RATIO during substep 2 of TSA.